

EXPONENTS

In the expression 5^2 , 5 is the base and 2 is the exponent. For x^a , x is the base and a is the exponent. 5^2 means $5 \cdot 5$ and 5^3 means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) or you can write it like this: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$.

You can use the Giant One to find the numbers in common. There are two Giant Ones, namely, $\frac{5}{5}$ twice, so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$. The Giant One here is $\frac{x}{x}$ (three of them). The answer is x^4 .

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$, which is 5^5 . $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$, which is 5^6 .

When the problems have variables such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is x^9 . If the problem is $(x^4)^5$ (x^4 to the fifth power) it means $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$. The answer is x^{20} . You multiply exponents in this case.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent ($10 - 4$).

The answer is x^6 . You can also have problems like $\frac{x^{10}}{x^{-4}}$. You still subtract, $10 - (-4)$ is 14, and the answer is x^{14} .

You need to be sure the bases are the same to use these laws. $x^5 \cdot y^6$ cannot be further simplified.

In general the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}$$

$$(x^a y^b)^c = x^{ac} y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see Math Notes box in Lesson 8.2.4 of the *Core Connections, Course 3* text.

Examples

a. $x^8 \cdot x^7 = x^{15}$

b. $\frac{x^{19}}{x^{13}} = x^6$

c. $(z^8)^3 = z^{24}$

d. $(x^2y^3)^4 = x^8y^{12}$

e. $\frac{x^4}{x^{-3}} = x^7$

f. $(2x^2y^3)^2 = 4x^4y^6$

g. $(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y^6}$

h. $\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

i. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

j. $5^2 \cdot 5^{-4} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Problems

Simplify each expression.

1. $5^2 \cdot 5^4$

2. $x^3 \cdot x^4$

3. $\frac{5^{16}}{5^{14}}$

4. $\frac{x^{10}}{x^6}$

5. $(5^3)^3$

6. $(x^4)^3$

7. $(4x^2y^3)^4$

8. $\frac{5^2}{5^{-3}}$

9. $5^5 \cdot 5^{-2}$

10. $(y^2)^{-3}$

11. $(4a^2b^{-2})^3$

12. $\frac{x^5y^4z^2}{x^4y^3z^2}$

13. $\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$

14. $4x^2 \cdot 2x^3$

15. 4^{-2}

16. 3^{-3}

17. $6^3 \cdot 6^{-2}$

18. $(3^{-1})^2$

Answers

1. 5^6

2. x^7

3. 5^2

4. x^4

5. 5^9

6. x^{12}

7. $256x^8y^{12}$

8. 5^5

9. 5^3

10. y^{-6} or $\frac{1}{y^6}$

11. $64a^6b^{-6}$ or $\frac{64a^6}{b^6}$

12. xy

13. $\frac{x^8z^4}{y}$ or $x^8y^{-1}z^4$

14. $8x^5$

15. $\frac{1}{16}$

16. $\frac{1}{27}$

17. 6

18. $\frac{1}{9}$

SCIENTIFIC NOTATION

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

Scientific Notation	Standard Form
5.32×10^{11}	532,000,000,000
2.61×10^{-15}	0.00000000000000261

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number 5.32×10^{11} means $5.32 \times 100,000,000,000$. Thus, two of the 11 places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example you are moving the decimal point to the right 11 places to find standard form.

The number 2.61×10^{-15} means $2.61 \times 0.000000000000001$. You are moving the decimal point to the left 15 places to find standard form. Here the standard form is 0.00000000000000261.

For additional information, see the Math Notes box in Lesson 8.2.3 of the *Core Connections, Course 3* text.

Example 1

Write each number in standard form.

$$7.84 \times 10^8 \Rightarrow 784,000,000 \quad \text{and} \quad 3.72 \times 10^{-3} \Rightarrow 0.00372$$

When taking a number in standard form and writing it in scientific notation, remember there is only *one* digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

Example 2

$$52,050,000 \Rightarrow 5.205 \times 10^7 \quad \text{and} \quad 0.000372 \Rightarrow 3.72 \times 10^{-4}$$

The exponent denotes the number of places you move the decimal point in the standard form. In the first example above, the decimal point is at the end of the number and it was moved 7 places. In the second example above, the exponent is negative because the original number is very small, that is, less than one.

Problems

Write each number in standard form.

1. 7.85×10^{11} 2. 1.235×10^9 3. 1.2305×10^3 4. 3.89×10^{-7} 5. 5.28×10^{-4}

Write each number in scientific notation.

6. 391,000,000,000 7. 0.0000842 8. 123056.7 9. 0.000000502
10. 25.7 11. 0.035 12. 5,600,000 13. 1346.8
14. 0.000000000006 15. 634,700,000,000,000

Note: On your scientific calculator, displays like 4.357^{12} and 3.65^{-3} are numbers expressed in scientific notation. The first number means 4.357×10^{12} and the second means 3.65×10^{-3} . The calculator does this because there is not enough room on its display window to show the entire number.

Answers

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|--------------------------|---------------------------|----------------------------|
| 1. 785,000,000,000 | 2. 1,235,000,000 | 3. 1230.5 |
| 4. 0.000000389 | 5. 0.000528 | 6. 3.91×10^{11} |
| 7. 8.42×10^{-5} | 8. 1.230567×10^5 | 9. 5.02×10^{-7} |
| 10. 2.57×10^1 | 11. 3.5×10^{-2} | 12. 5.6×10^6 |
| 13. 1.3468×10^3 | 14. 6.0×10^{-12} | 15. 6.347×10^{14} |