

Simple interest is paid only on the original amount invested. The formula for simple interest is $I = Prt$ and the total amount including interest would be $A = P + I$. In *Core Connections, Course 3*, students are introduced to compound interest using the formula $A = P(1 + r)^n$. Compound interest is paid on both the original amount invested and the interest previously earned. Note that in these formulas, P = principal (amount invested), r = rate of interest, t and n both represent the number of time periods for which the total amount, A , is calculated and I = interest earned.

For additional information, see the Math Notes box in Lesson 8.1.3 of the *Core Connections, Course 3* text.

Example 1

Wayne earns 5.3% simple interest for 5 years on \$3000. How much interest does he earn and what is the total amount in the account?

Put the numbers in the formula $I = Prt$.	$I = 3000(5.3\%)5$
Change the percent to a decimal.	$= 3000(0.053)5$
Multiply.	$= 795$ Wayne would earn \$795 interest.
Add principal and interest.	$\$3000 + \$795 = \$3795$ in the account

Example 2

Use the numbers in Example 1 to find how much money Wayne would have if he earned 5.3% interest compounded annually.

Put the numbers in the formula $A = P(1 + r)^n$.	$A = 3000(1 + 5.3\%)^5$
Change the percent to a decimal.	$= 3000(1 + 0.053)^5$ or $3000(1.053)^5$
Multiply.	$= 3883.86$

Wayne would have \$3883.86.

Students are asked to compare the difference in earnings when an amount is earning simple or compound interest. In these examples, Wayne would have \$88.86 more with compound interest than he would have with simple interest: $\$3883.86 - \$3795 = \$88.86$.

Problems

Solve the following problems.

1. Tong loaned Jody \$50 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?
2. Jessica's grandparents gave her \$2000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the \$2000 for every year. How much extra money will her grandparents give her at the end of four years?
3. David read an ad offering $8\frac{3}{4}\%$ simple interest on accounts over \$500 left for a minimum of 5 years. He has \$500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?
4. Javier's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Javier was 15, the account had a total of \$1012.50 interest paid on it. How much did Javier's parents set aside when he was born?
5. Kristina received \$125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?
6. Kristina decided she would do better if she put her money in the bank, which paid 2.8% interest compounded annually. Was she right?
7. Suppose Jessica (from problem 2) had put her \$2000 in the bank at 3.25% interest compounded annually. How much money would she have earned there at the end of 4 years?
8. Mai put \$4250 in the bank at 4.4% interest compounded annually. How much was in her account after 7 years?
9. What is the difference in the amount of money in the bank after five years if \$2500 is invested at 3.2% interest compounded annually or at 2.9% interest compounded annually?
10. Ronna was listening to her parents talking about what a good deal compounded interest was for a retirement account. She wondered how much money she would have if she invested \$2000 at age 20 at 2.8% annual interest compounded quarterly (four times each year) and left it until she reached age 65. Determine what the value of the \$2000 would become.

Answers

1. $I = 50(0.05)1 = \$2.50$; Jody paid back \$52.50.
2. $I = 2000(0.075)4 = \$600$
3. $I = \$500(0.0875)5 = \218.75
4. $\$1012.50 = x(0.045)15$; $x = \$1500$
5. $I = 125(0.035)1 = \$4.38$
6. $A = 125(1 + 0.028)^1 = \$128.50$; No, for one year she needs to take the higher interest rate if the compounding is done annually. Only after one year will compounding earn more than simple interest.
7. $A = 2000(1 + 0.0325)^4 = \2272.95
8. $A = 4250(1 + 0.044)^7 = \5745.03
9. $A = 2500(1 + 0.032)^5 - 2500(1 + 0.029)^5 = \$2926.43 - \$2884.14 = \42.29
10. $A = 2000(1 + 0.007)^{180}$ (because $45 \cdot 4 = 180$ quarters) = \$7019.96

EXPONENTS

In the expression 5^2 , 5 is the base and 2 is the exponent. For x^a , x is the base and a is the exponent. 5^2 means $5 \cdot 5$ and 5^3 means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) or you can write it like this: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$.

You can use the Giant One to find the numbers in common. There are two Giant Ones, namely, $\frac{5}{5}$ twice, so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$. The Giant One here is $\frac{x}{x}$ (three of them). The answer is x^4 .

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$, which is 5^5 . $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$, which is 5^6 .

When the problems have variables such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is x^9 . If the problem is $(x^4)^5$ (x^4 to the fifth power) it means $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$. The answer is x^{20} . You multiply exponents in this case.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent ($10 - 4$).

The answer is x^6 . You can also have problems like $\frac{x^{10}}{x^{-4}}$. You still subtract, $10 - (-4)$ is 14, and the answer is x^{14} .

You need to be sure the bases are the same to use these laws. $x^5 \cdot y^6$ cannot be further simplified.

In general the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1$$

$$x^{-n} = \frac{1}{x^n}$$

$$(x^a y^b)^c = x^{ac} y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see Math Notes box in Lesson 8.2.4 of the *Core Connections, Course 3* text.

Examples

a. $x^8 \cdot x^7 = x^{15}$

b. $\frac{x^{19}}{x^{13}} = x^6$

c. $(z^8)^3 = z^{24}$

d. $(x^2y^3)^4 = x^8y^{12}$

e. $\frac{x^4}{x^{-3}} = x^7$

f. $(2x^2y^3)^2 = 4x^4y^6$

g. $(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y^6}$

h. $\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

i. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

j. $5^2 \cdot 5^{-4} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Problems

Simplify each expression.

1. $5^2 \cdot 5^4$

2. $x^3 \cdot x^4$

3. $\frac{5^{16}}{5^{14}}$

4. $\frac{x^{10}}{x^6}$

5. $(5^3)^3$

6. $(x^4)^3$

7. $(4x^2y^3)^4$

8. $\frac{5^2}{5^{-3}}$

9. $5^5 \cdot 5^{-2}$

10. $(y^2)^{-3}$

11. $(4a^2b^{-2})^3$

12. $\frac{x^5y^4z^2}{x^4y^3z^2}$

13. $\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$

14. $4x^2 \cdot 2x^3$

15. 4^{-2}

16. 3^{-3}

17. $6^3 \cdot 6^{-2}$

18. $(3^{-1})^2$

Answers

1. 5^6

2. x^7

3. 5^2

4. x^4

5. 5^9

6. x^{12}

7. $256x^8y^{12}$

8. 5^5

9. 5^3

10. y^{-6} or $\frac{1}{y^6}$

11. $64a^6b^{-6}$ or $\frac{64a^6}{b^6}$

12. xy

13. $\frac{x^8z^4}{y}$ or $x^8y^{-1}z^4$

14. $8x^5$

15. $\frac{1}{16}$

16. $\frac{1}{27}$

17. 6

18. $\frac{1}{9}$

SCIENTIFIC NOTATION

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

Scientific Notation	Standard Form
5.32×10^{11}	532,000,000,000
2.61×10^{-15}	0.00000000000000261

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number 5.32×10^{11} means $5.32 \times 100,000,000,000$. Thus, two of the 11 places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example you are moving the decimal point to the right 11 places to find standard form.

The number 2.61×10^{-15} means $2.61 \times 0.000000000000001$. You are moving the decimal point to the left 15 places to find standard form. Here the standard form is 0.00000000000000261.

For additional information, see the Math Notes box in Lesson 8.2.3 of the *Core Connections, Course 3* text.

Example 1

Write each number in standard form.

$$7.84 \times 10^8 \Rightarrow 784,000,000 \quad \text{and} \quad 3.72 \times 10^{-3} \Rightarrow 0.00372$$

When taking a number in standard form and writing it in scientific notation, remember there is only *one* digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

Example 2

$$52,050,000 \Rightarrow 5.205 \times 10^7 \quad \text{and} \quad 0.000372 \Rightarrow 3.72 \times 10^{-4}$$

The exponent denotes the number of places you move the decimal point in the standard form. In the first example above, the decimal point is at the end of the number and it was moved 7 places. In the second example above, the exponent is negative because the original number is very small, that is, less than one.

Problems

Write each number in standard form.

1. 7.85×10^{11} 2. 1.235×10^9 3. 1.2305×10^3 4. 3.89×10^{-7} 5. 5.28×10^{-4}

Write each number in scientific notation.

6. 391,000,000,000 7. 0.0000842 8. 123056.7 9. 0.000000502
10. 25.7 11. 0.035 12. 5,600,000 13. 1346.8
14. 0.0000000000006 15. 634,700,000,000,000

Note: On your scientific calculator, displays like 4.357^{12} and 3.65^{-3} are numbers expressed in scientific notation. The first number means 4.357×10^{12} and the second means 3.65×10^{-3} . The calculator does this because there is not enough room on its display window to show the entire number.

Answers

1. 785,000,000,000 2. 1,235,000,000 3. 1230.5
4. 0.000000389 5. 0.000528 6. 3.91×10^{11}
7. 8.42×10^{-5} 8. 1.230567×10^5 9. 5.02×10^{-7}
10. 2.57×10^1 11. 3.5×10^{-2} 12. 5.6×10^6
13. 1.3468×10^3 14. 6.0×10^{-12} 15. 6.347×10^{14}