RIGID TRANSFORMATIONS 6.1.1 – 6.1.4

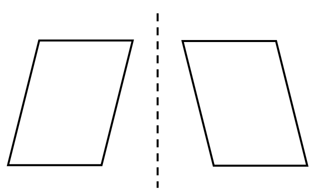
Studying transformations of geometric shapes builds a foundation for a key idea in geometry: congruence. In this introduction to transformations, the students explore three rigid motions: translations, reflections, and rotations. A translation slides a figure horizontally, vertically or both. A reflection flips a figure across a fixed line (for example, the x-axis). A rotation turns an object about a point (for example, $(0, 0)$). This exploration is done with simple tools that can be found at home (tracing paper) as well as with computer software. Students change the position and/or orientation of a shape by applying one or more of these motions to the original figure to create its image in a new position without changing its size or shape. Transformations also lead directly to studying symmetry in shapes. These ideas will help with describing and classifying geometric shapes later in the course.

For additional information, see the Math Notes box in Lesson 6.1.3 of the *Core Connections, Course 3* text.

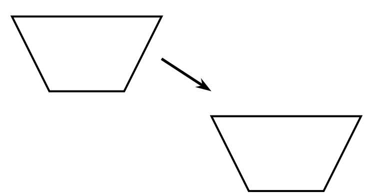
Example 1

Decide which transformation was used on each pair of shapes below. Some may be a combination of transformations.

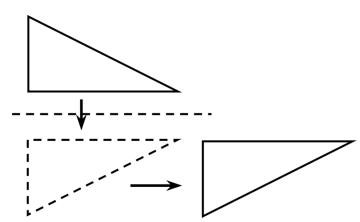
a.



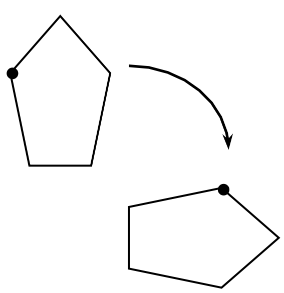
b.



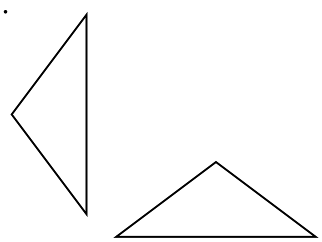
c.



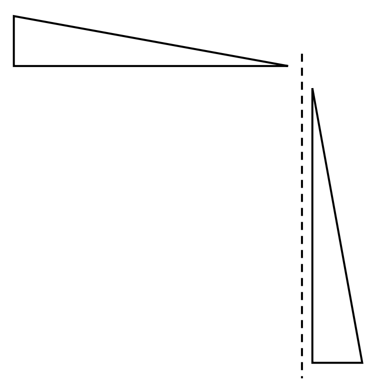
d.



e.



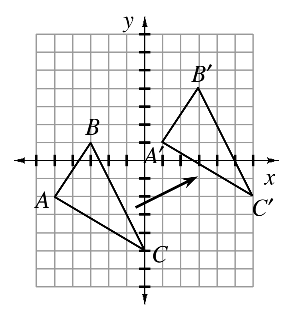
f.



Identifying a single transformation is usually easy for students. In part (a), the parallelogram is reflected (flipped) across an invisible vertical line. (Imagine a mirror running vertically between the two figures. One figure would be the reflection of the other.) Reflecting a shape once changes its orientation, that is, how its parts "sit" on the flat surface. For example, in part (a), the two sides of the figure at left slant upwards to the right, whereas in its reflection at right, they slant upwards to the left. Likewise, the angles in the figure at left "switch positions" in the figure at right.

In part (b), the shape is translated (or slid) to the right and down. The orientation is the same. Part (c) shows a combination of transformations. First the triangle is reflected (flipped) across an invisible horizontal line. Then it is translated (slid) to the right. The pentagon in part (d) has been rotated (turned) clockwise to create the second figure. Imagine tracing the first figure on tracing paper, then holding the tracing paper with a pin at one point below the first pentagon, then turning the paper to the right (that is, clockwise) 90º. The second pentagon would be the result. Some students might see this as a reflection across a diagonal line. The pentagon itself could be, but with the added dot, the entire shape cannot be a reflection. If it had been reflected, the dot would have to be on the corner below the one shown in the rotated figure. The triangles in part (e) are rotations of each other (90º clockwise again). Part (f) shows another combination. The triangle is rotated (the horizontal side becomes vertical) but also reflected since the longest side of the triangle points in the opposite direction from the first figure.

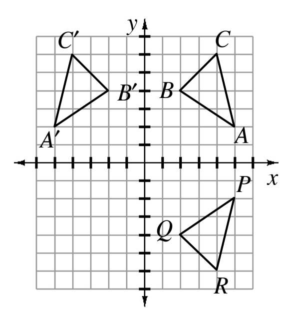
Example 2



Translate (slide) ΔABC right six units and up three units. Give the coordinates of the vertices of the new triangle.

The original vertices are $A(-5,-2)$, $B(-3, 1)$, and $C(0,-5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6,-2)$. Notice that the change to each original point (x, y) can be represented by $(x + 6, y + 3)$.

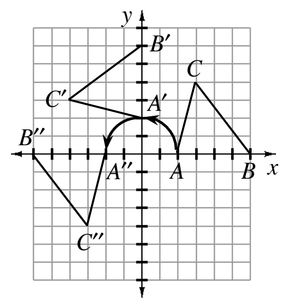
Example 3



Reflect (flip) ΔABC with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the y-axis to get $\Delta A'B'C'$. The key is that the reflection is the same distance from the y-axis as the original figure. The new points are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the y-axis, the change to each original point (x, y) can be represented by $(-x, y)$.

If you reflect ΔABC across the x-axis to get ΔPQR, then the new points are $P(5,-2)$, $Q(2,-4)$, and $R(4,-6)$. In this case, reflecting across the x-axis, the change to each original point (x,y) can be represented by $(x,-y)$.

Example 4



Rotate (turn) ΔABC with coordinates $A(2,0)$, $B(6,0)$, and $C(3,4)$ 90° counterclockwise about the origin $(0,0)$ to get $\Delta A'B'C'$ with coordinates $A'(0,2)$, $B'(0,6)$, and $C'(-4,3)$. Notice that for this 90° counterclockwise rotation about the origin, the change to each original point (x,y) can be represented by $(-y,x)$.

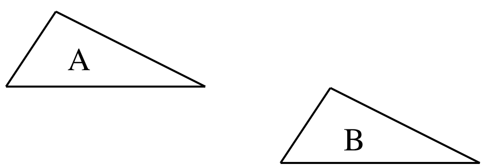
Rotating another 90° (180° from the starting location) yields $\Delta A''B''C''$ with coordinates $A''(-2,0)$, $B''(-6,0)$, and $C''(-3,-4)$.

For this 180° counterclockwise rotation about the origin, the change to each original point (x,y) can be represented by $(-x,-y)$. Similarly a 270° counterclockwise or 90° clockwise rotation about the origin takes each original point (x,y) to the point $(y,-x)$.

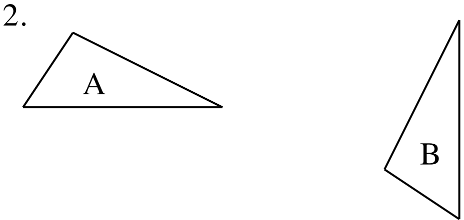
Problems

For each pair of triangles, describe the transformation that moves triangle A to the location of triangle B.

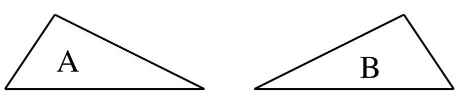
1.



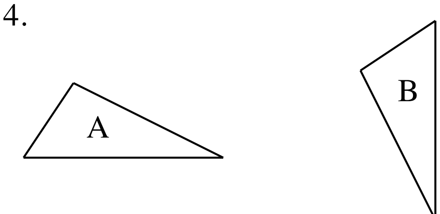
2.



3.



4.



For the following problems, refer to the figures below:

Figure A

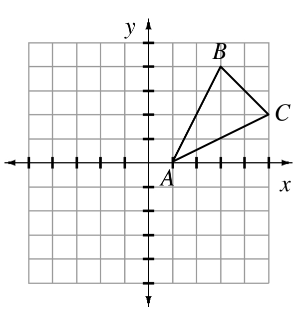


Figure B

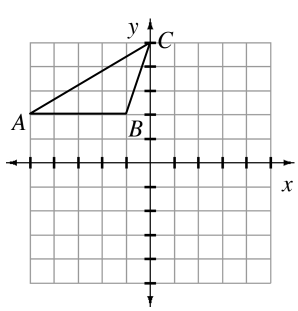
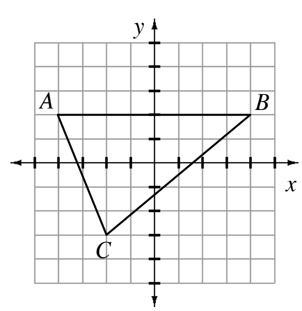


Figure C



State the new coordinates after each transformation.

5. Slide figure A left 2 units and down 3 units.

6. Slide figure B right 3 units and down 5 units.

7. Slide figure C left 1 unit and up 2 units.

8. Flip figure A across the *x*-axis.

9. Flip figure B across the *x*-axis.

10. Flip figure C across the *x*-axis.

11. Flip figure A across the *y*-axis.

12. Flip figure B across the *y*-axis.

13. Flip figure C across the *y*-axis.

14. Rotate figure A 90° counterclockwise about the origin.

15. Rotate figure B 90° counterclockwise about the origin.

16. Rotate figure C 90° counterclockwise about the origin.

17. Rotate figure A 180° counterclockwise about the origin.

18. Rotate figure C 180° counterclockwise about the origin.

19. Rotate figure B 270° counterclockwise about the origin.

20. Rotate figure C 90° clockwise about the origin.

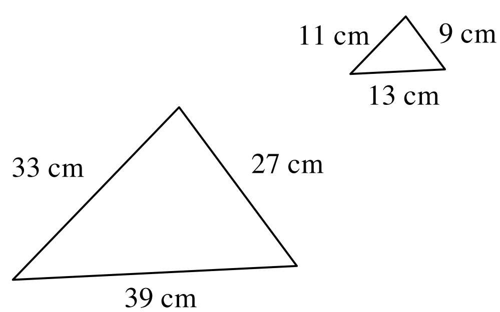
Answers (1 to 4 may vary; 5 to 20 given in the order A', B', C')

1.	translation	2.	rotation and translation
3.	reflection	4.	rotation and translation
5.	(–1, –3), (1, 2), (3, –1)	6.	(–2, –3), (2, –3), (3, 0)
7.	(–5, 4), (3, 4), (–3, –1)	8.	(1, 0), (3, –4), (5, –2)
9.	(–5, –2), (–1, –2), (0, –5)	10.	(–4, –2), (4, –2), (–2, 3)
11.	(–1, 0), (–3, 4), (–5, 2)	12.	(5, 2), (1, 2), (0, 5)
13.	(4, 2), (–4, 2), (2, –3)	14.	(0, 1), (–4, 3), (–2, 5)
15.	(–2, –5), (–5, 0), (–2, –1)	16.	(–2, –4), (–2, 4), (3, –2)
17.	(–1, 0), (–3, –4), (–5, –2)	18.	(4, –2), (–4, –2), (2, 3)
19.	(2, 5), (2, 1), (5, 0)	20.	(2, 4), (2, –4), (–3, 2)

Two figures that have the same shape but not necessarily the same size are similar. In similar figures the measures of the corresponding angles are equal and the ratios of the corresponding sides are proportional. This ratio is called the scale factor. For information about corresponding sides and angles of similar figures see the Math Notes box in Lesson 6.2.2 of the *Core Connections, Course 3* text. For information about scale factor and similarity, see the Math Notes box in Lesson 6.2.6 of the *Core Connections, Course 3* text.

Example 1

Determine if the figures are similar. If so, what is the scale factor?

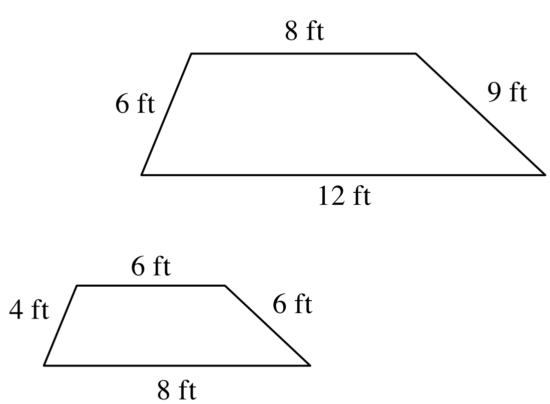


$\frac{39}{13} = \frac{33}{11} = \frac{27}{9} = \frac{3}{1}$ or 3

The ratios of corresponding sides are equal so the figures are similar. The scale factor that compares the small figure to the large one is 3 or 3 to 1. The scale factor that compares the large figure to the small figure is $\frac{1}{3}$ or 1 to 3.

Example 2

Determine if the figures are similar. If so, state the scale factor.

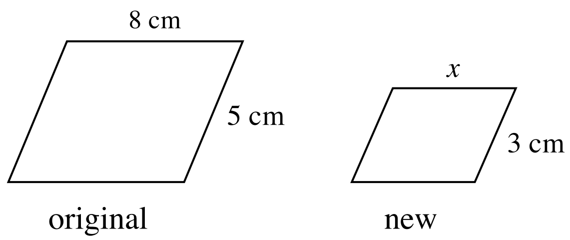


$\frac{6}{4} = \frac{12}{8} = \frac{9}{6}$ and all equal $\frac{3}{2}$.

$\frac{8}{6} = \frac{4}{3}$ so the shapes are not similar.

Example 3

Determine the scale factor for the pair of similar figures. Use the scale factor to find the side length labeled with a variable.



scale factor = $\frac{3}{5}$

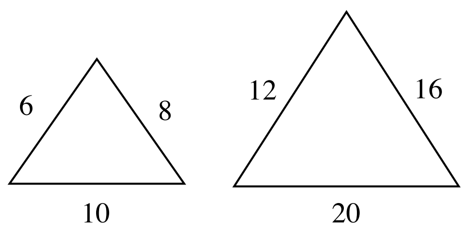
original $\cdot \frac{3}{5} \Rightarrow$ new

$8 \cdot \frac{3}{5} = x\,;\ \Rightarrow\ x = \frac{24}{5} = 4.8$ cm

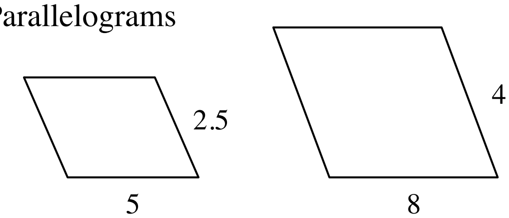
Problems

Determine if the figures are similar. If so, state the scale factor of the first to the second.

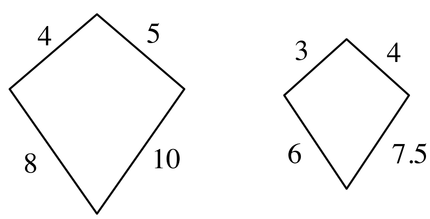
1.



2. Parallelograms

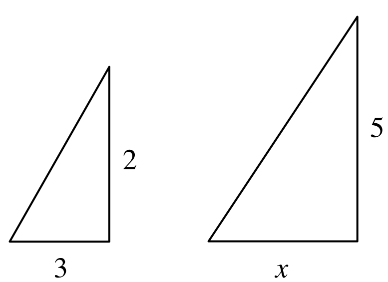


3.

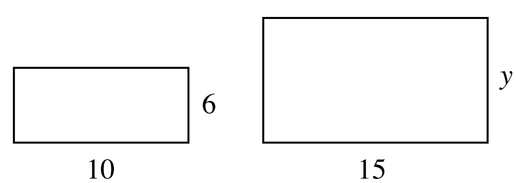


Determine the scale factor for each pair of similar figures. Use the scale factor to find the side labeled with the variable.

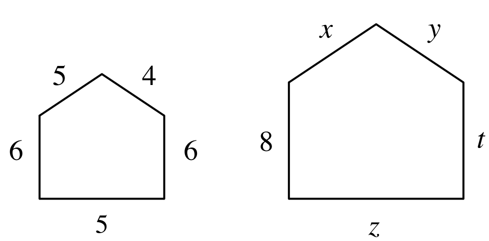
4.



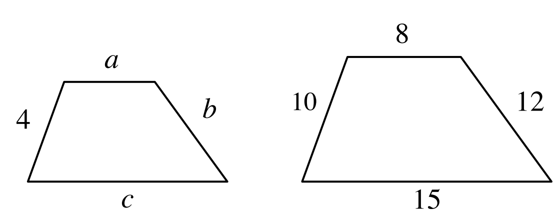
5.



6.



7.



Answers

1. similar; 2

2. similar; $\frac{8}{5} = 1.6$

3. not similar

4. $\frac{5}{2}$; $x = 7.5$

5. $\frac{3}{2}$; $y = 9$

6. $\frac{4}{3}$; $x = \frac{20}{3} = 6\frac{2}{3}$, $y = \frac{16}{3} = 5\frac{1}{3}$, $t = 8$, $z = \frac{25}{3} = 8\frac{1}{3}$

7. $\frac{5}{2}$; $a = \frac{16}{5} = 3.2$, $b = \frac{24}{5} = 4.8$, $c = 6$

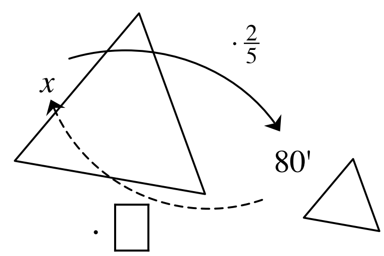
SCALING TO SOLVE PERCENT AND OTHER PROBLEMS 6.2.4 – 6.2.6

Students used scale factors (multipliers) to enlarge and reduce figures as well as increase and decrease quantities. All of the original quantities or lengths were multiplied by the scale factor to get the new quantities or lengths. To reverse this process and scale from the new situation back to the original, we divide by the scale factor. Division by a scale factor is the same as multiplying by a reciprocal. This same concept is useful in solving equations with fractional coefficients. To remove a fractional coefficient you may divide each term in the equation by the coefficient or multiply each term by the reciprocal of the coefficient. Recall that a reciprocal is the multiplicative inverse of a number, that is, the product of the two numbers is 1. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, $\frac{1}{2}$ is $\frac{2}{1}$, and 5 is $\frac{1}{5}$.

Scaling may also be used with percentage problems where a quantity is increased or decreased by a certain percent. Scaling by a factor of 1 does not change the quantity. Increasing by a certain percent may be found by multiplying by (1 + the percent) and decreasing by a certain percent may be found by multiplying by (1 – the percent).

Example 1

The large triangle at right was reduced by a scale factor of $\frac{2}{5}$ to create a similar triangle. If the side labeled x now has a length of 80' in the new figure, what was the original length?



To undo the reduction, multiply 80' by the reciprocal of $\frac{2}{5}$, namely $\frac{5}{2}$, or divide 80' by $\frac{2}{5}$.

$80' \div \frac{2}{5}$ is the same as $80' \cdot \frac{5}{2}$, so $x = 200'$.

Example 2

Solve: $\frac{2}{3}x = 12$

Method 1: Use division and a Giant One

$$\frac{2}{3}x = 12$$

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{12}{\frac{2}{3}}$$

$$x = \frac{12}{\frac{2}{3}} = 12 \div \frac{2}{3} = \frac{36}{3} \div \frac{2}{3} = \frac{36}{2} = 18$$

Method 2: Use reciprocals

$$\frac{2}{3}x = 12$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(12)$$

$$x = 18$$

Example 3

Samantha wants to leave a 15% tip on her lunch bill of $12.50. What scale factor should be used and how much money should she leave?

Since tipping increases the total, the scale factor is (1 + 15%) = 1.15.
She should leave (1.15)(12.50) = $14.38 or about $14.50.

Example 4

Carlos sees that all DVDs are on sales at 40% off. If the regular price of a DVD is $24.95, what is the scale factor and how much is the sale price?

If items are reduced 40%, the scale factor is (1 – 40%) = 0.60.
The sale price is (0.60)(24.95) = $14.97.

Problems

1. A rectangle was enlarged by a scale factor of $\frac{5}{2}$ and the new width is 40 cm. What was the original width?

2. A side of a triangle was reduced by a scale factor of $\frac{2}{3}$. If the new side is now 18 inches, what was the original side?

3. The scale factor used to create the design for a backyard is 2 inches for every 75 feet ($\frac{2}{75}$). If on the design, the fire pit is 0.5 inches away from the house, how far from the house, in feet, should the fire pit be dug?

4. After a very successful year, Cheap-Rentals raised salaries by a scale factor of $\frac{11}{10}$. If Luan now makes $14.30 per hour, what did she earn before?

5. Solve: $\frac{3}{4}x = 60$

6. Solve: $\frac{2}{5}x = 42$

7. Solve: $\frac{3}{5}y = 40$

8. Solve: $-\frac{8}{3}m = 6$

9. What is the total cost of a $39.50 family dinner after you add a 20% tip?

10. If the current cost to attend Magicland Park is now $29.50 per person, what will be the cost after a 8% increase?

11. Winter coats are on clearance at 60% off. If the regular price is $79, what is the sale price?

12. The company president has offered to reduce her salary 10% to cut expenses. If she now earns $175,000, what will be her new salary?

Answers

1.	16 cm	2.	27 inches	3.	$18\frac{3}{4}$ feet	4.	\$13.00
5.	80	6.	105	7.	$66\frac{2}{3}$	8.	$-2\frac{1}{4}$
9.	\$47.40	10.	\$31.86	11.	\$31.60	12.	\$157,500