

# COMMON CORE STATE STANDARDS FOR

---

## Mathematics



# Table of Contents

<b>Introduction</b>	<b>3</b>
<b>Standards for Mathematical Practice</b>	<b>6</b>
<b>Standards for Mathematical Content</b>	
Kindergarten	<b>9</b>
Grade 1	<b>13</b>
Grade 2	<b>17</b>
Grade 3	<b>21</b>
Grade 4	<b>27</b>
Grade 5	<b>33</b>
Grade 6	<b>39</b>
Grade 7	<b>46</b>
Grade 8	<b>52</b>
High School — Introduction	
High School — Number and Quantity	<b>58</b>
High School — Algebra	<b>62</b>
High School — Functions	<b>67</b>
High School — Modeling	<b>72</b>
High School — Geometry	<b>74</b>
High School — Statistics and Probability	<b>79</b>
<b>Glossary</b>	<b>85</b>
<b>Sample of Works Consulted</b>	<b>91</b>

# Introduction

## Toward greater focus and coherence

*Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*

— Mathematics Learning in Early Childhood, National Research Council, 2009

*The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K-6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1-3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*

— Ginsburg, Leinwand and Decker, 2009

*Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*

— Ginsburg et al., 2005

*There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*

— Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

*articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies*

*that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students...absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

### **Understanding mathematics**

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as  $(a + b)(x + y)$  and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding  $(a + b + c)(x + y)$ . Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

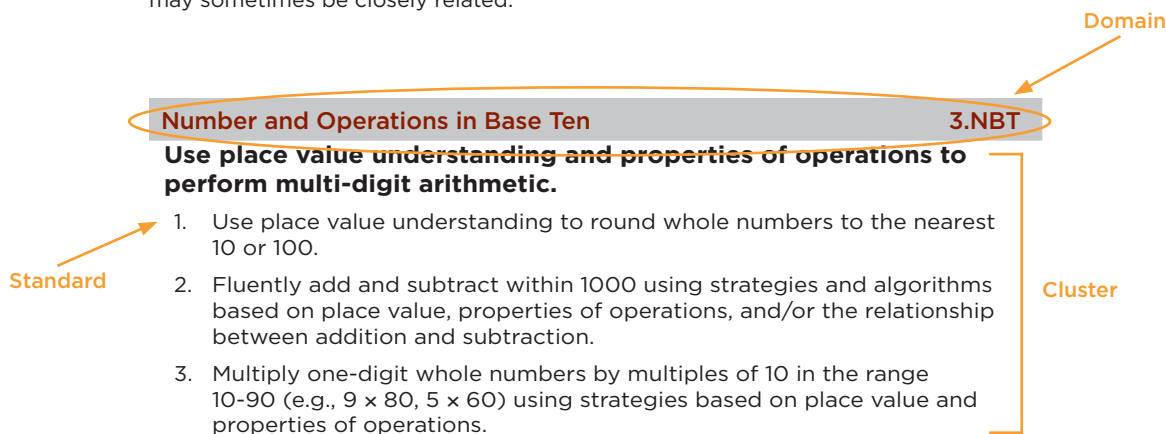
The Standards begin on page 6 with eight Standards for Mathematical Practice.

## How to read the grade level standards

**Standards** define what students should understand and be able to do.

**Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.



These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know ... should next come to learn ....” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

# Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.



## Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as  $5 + 2 = 7$  and  $7 - 2 = 5$ . (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

# Grade K Overview

## Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

## Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

## Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

## Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

## Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Counting and Cardinality****K.CC****Know number names and the count sequence.**

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

**Count to tell the number of objects.**

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
  - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
  - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  - c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

**Compare numbers.**

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>
7. Compare two numbers between 1 and 10 presented as written numerals.

**Operations and Algebraic Thinking****K.OA****Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

1. Represent addition and subtraction with objects, fingers, mental images, drawings<sup>2</sup>, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

<sup>1</sup>Include groups with up to ten objects.

<sup>2</sup>Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

**Number and Operations in Base Ten****K.NBT****Work with numbers 11–19 to gain foundations for place value.**

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g.,  $18 = 10 + 8$ ); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

**Measurement and Data****K.MD****Describe and compare measurable attributes.**

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

**Classify objects and count the number of objects in each category.**

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.<sup>3</sup>

**Geometry****K.G****Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

**Analyze, compare, create, and compose shapes.**

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”*

<sup>3</sup>Limit category counts to be less than or equal to 10.

## Glossary

**Addition and subtraction within 5, 10, 20, 100, or 1000.** Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example:  $8 + 2 = 10$  is an addition within 10,  $14 - 5 = 9$  is a subtraction within 20, and  $55 - 18 = 37$  is a subtraction within 100.

**Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another. Example:  $\frac{3}{4}$  and  $-\frac{3}{4}$  are additive inverses of one another because  $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$ .

**Associative property of addition.** See Table 3 in this Glossary.

**Associative property of multiplication.** See Table 3 in this Glossary.

**Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

**Box plot.** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.<sup>1</sup>

**Commutative property.** See Table 3 in this Glossary.

**Complex fraction.** A fraction  $\frac{A}{B}$  where  $A$  and/or  $B$  are fractions ( $B$  nonzero).

**Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See *also*: computation strategy.

**Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See *also*: computation algorithm.

**Congruent.** Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

**Counting on.** A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

**Dot plot.** See: line plot.

**Dilation.** A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

**Expanded form.** A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example,  $643 = 600 + 40 + 3$ .

**Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

**First quartile.** For a data set with median  $M$ , the first quartile is the median of the data values less than  $M$ . Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the first quartile is 6.<sup>2</sup> See *also*: median, third quartile, interquartile range.

**Fraction.** A number expressible in the form  $\frac{a}{b}$  where  $a$  is a whole number and  $b$  is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) See *also*: rational number.

**Identity property of 0.** See Table 3 in this Glossary.

**Independently combined probability models.** Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

<sup>1</sup>Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/mathglos.html>, accessed March 2, 2010.

<sup>2</sup>Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

**Integer.** A number expressible in the form  $a$  or  $-a$  for some whole number  $a$ .

**Interquartile Range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is  $15 - 6 = 9$ . See *also*: first quartile, third quartile.

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.<sup>3</sup>

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.<sup>4</sup> Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

**Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example:  $72 \div 8 = 9$ .

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. Example:  $\frac{3}{4}$  and  $\frac{4}{3}$  are multiplicative inverses of one another because  $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$ .

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

**Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by  $\frac{5}{50} = 10\%$  per year.

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

**Properties of operations.** See Table 3 in this Glossary.

**Properties of equality.** See Table 4 in this Glossary.

**Properties of inequality.** See Table 5 in this Glossary.

**Properties of operations.** See Table 3 in this Glossary.

**Probability.** A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See *also*: uniform probability model.

**Random variable.** An assignment of a numerical value to each outcome in a sample space.

**Rational expression.** A quotient of two polynomials with a non-zero denominator.

**Rational number.** A number expressible in the form  $\frac{a}{b}$  or  $-\frac{a}{b}$  for some fraction  $\frac{a}{b}$ . The rational numbers include the integers.

**Rectilinear figure.** A polygon all angles of which are right angles.

**Rigid motion.** A transformation of points in space consisting of a sequence of

<sup>3</sup>Adapted from Wisconsin Department of Public Instruction, *op. cit.*

<sup>4</sup>To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

**Repeating decimal.** The decimal form of a rational number. *See also:* terminating decimal.

**Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered.

**Scatter plot.** A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.<sup>5</sup>

**Similarity transformation.** A rigid motion followed by a dilation.

**Tape diagram.** A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

**Terminating decimal.** A decimal is called terminating if its repeating digit is 0.

**Third quartile.** For a data set with median  $M$ , the third quartile is the median of the data values greater than  $M$ . Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

**Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

**Uniform probability model.** A probability model which assigns equal probability to all outcomes. *See also:* probability model.

**Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

**Visual fraction model.** A tape diagram, number line diagram, or area model.

**Whole numbers.** The numbers 0, 1, 2, 3, ....

---

<sup>5</sup>Adapted from Wisconsin Department of Public Instruction, *op. cit.*

TABLE 1. Common addition and subtraction situations.<sup>6</sup>

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>1</sup>
<b>Put Together/ Take Apart<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
<b>Compare<sup>3</sup></b>	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?  (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?  (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?  (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

<sup>1</sup>These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

<sup>2</sup>Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

<sup>3</sup>For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

<sup>6</sup>Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).



TABLE 2. Common multiplication and division situations.<sup>7</sup>

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ , and $18 \div 3 = ?$	$? \times 6 = 18$ , and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays, <sup>4</sup> Area <sup>5</sup>	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

<sup>4</sup>The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>5</sup>Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

<sup>7</sup>The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

**TABLE 3.** The properties of operations. Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

<i>Associative property of addition</i>	$(a + b) + c = a + (b + c)$
<i>Commutative property of addition</i>	$a + b = b + a$
<i>Additive identity property of 0</i>	$a + 0 = 0 + a = a$
<i>Existence of additive inverses</i>	For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$ .
<i>Associative property of multiplication</i>	$(a \times b) \times c = a \times (b \times c)$
<i>Commutative property of multiplication</i>	$a \times b = b \times a$
<i>Multiplicative identity property of 1</i>	$a \times 1 = 1 \times a = a$
<i>Existence of multiplicative inverses</i>	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$ .
<i>Distributive property of multiplication over addition</i>	$a \times (b + c) = a \times b + a \times c$

**TABLE 4.** The properties of equality. Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in the rational, real, or complex number systems.

<i>Reflexive property of equality</i>	$a = a$
<i>Symmetric property of equality</i>	If $a = b$ , then $b = a$ .
<i>Transitive property of equality</i>	If $a = b$ and $b = c$ , then $a = c$ .
<i>Addition property of equality</i>	If $a = b$ , then $a + c = b + c$ .
<i>Subtraction property of equality</i>	If $a = b$ , then $a - c = b - c$ .
<i>Multiplication property of equality</i>	If $a = b$ , then $a \times c = b \times c$ .
<i>Division property of equality</i>	If $a = b$ and $c \neq 0$ , then $a \div c = b \div c$ .
<i>Substitution property of equality</i>	If $a = b$ , then $b$ may be substituted for $a$ in any expression containing $a$ .

**TABLE 5.** The properties of inequality. Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$ , $a = b$ , $a > b$ .
If $a > b$ and $b > c$ then $a > c$ .
If $a > b$ , then $b < a$ .
If $a > b$ , then $-a < -b$ .
If $a > b$ , then $a \pm c > b \pm c$ .
If $a > b$ and $c > 0$ , then $a \times c > b \times c$ .
If $a > b$ and $c < 0$ , then $a \times c < b \times c$ .
If $a > b$ and $c > 0$ , then $a \div c > b \div c$ .
If $a > b$ and $c < 0$ , then $a \div c < b \div c$ .

## Sample of Works Consulted

- Existing state standards documents.
- Research summaries and briefs provided to the Working Group by researchers.
- National Assessment Governing Board, *Mathematics Framework for the 2009 National Assessment of Educational Progress*. U.S. Department of Education, 2008.
- NAEP Validity Studies Panel, *Validity Study of the NAEP Mathematics Assessment: Grades 4 and 8*. Daro et al., 2007.
- Mathematics documents from: Alberta, Canada; Belgium; China; Chinese Taipei; Denmark; England; Finland; Hong Kong; India; Ireland; Japan; Korea; New Zealand; Singapore; Victoria (British Columbia).
- Adding it Up: Helping Children Learn Mathematics. National Research Council, Mathematics Learning Study Committee, 2001.
- Benchmarking for Success: Ensuring U.S. Students Receive a World-Class Education. National Governors Association, Council of Chief State School Officers, and Achieve, Inc., 2008.
- Crossroads in Mathematics* (1995) and *Beyond Crossroads* (2006). American Mathematical Association of Two-Year Colleges (AMATYC).
- Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. National Council of Teachers of Mathematics, 2006.
- Focus in High School Mathematics: Reasoning and Sense Making*. National Council of Teachers of Mathematics. Reston, VA: NCTM.
- Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. U.S. Department of Education: Washington, DC, 2008.
- Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK-12 Curriculum Framework*.
- How People Learn: Brain, Mind, Experience, and School*. Bransford, J.D., Brown, A.L., and Cocking, R.R., eds. Committee on Developments in the Science of Learning, Commission on Behavioral and Social Sciences and Education, National Research Council, 1999.
- Mathematics and Democracy, The Case for Quantitative Literacy*, Steen, L.A. (ed.). National Council on Education and the Disciplines, 2001.
- Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Cross, C.T., Woods, T.A., and Schweingruber, S., eds. Committee on Early Childhood Mathematics, National Research Council, 2009.
- The Opportunity Equation: Transforming Mathematics and Science Education for Citizenship and the Global Economy*. The Carnegie Corporation of New York and the Institute for Advanced Study, 2009. Online: <http://www.opportunityequation.org/>
- Principles and Standards for School Mathematics*. National Council of Teachers of Mathematics, 2000.
- The Proficiency Illusion*. Cronin, J., Dahlin, M., Adkins, D., and Kingsbury, G.G.; foreword by C.E. Finn, Jr., and M. J. Petrilli. Thomas B. Fordham Institute, 2007.
- Ready or Not: Creating a High School Diploma That Counts*. American Diploma Project, 2004.
- A Research Companion to Principles and Standards for School Mathematics*. National Council of Teachers of Mathematics, 2003.
- Sizing Up State Standards 2008*. American Federation of Teachers, 2008.
- A Splintered Vision: An Investigation of U.S. Science and Mathematics Education*. Schmidt, W.H., McKnight, C.C., Raizen, S.A., et al. U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University, 1997.
- Stars By Which to Navigate? Scanning National and International Education Standards in 2009*. Carmichael, S.B., Wilson, W.S., Finn, Jr., C.E., Winkler, A.M., and Palmieri, S. Thomas B. Fordham Institute, 2009.
- Askey, R., "Knowing and Teaching Elementary Mathematics," *American Educator*, Fall 1999.
- Aydogan, C., Plummer, C., Kang, S. J., Bilbrey, C., Farran, D. C., & Lipsey, M. W. (2005). An investigation of prekindergarten curricula: Influences on classroom characteristics and child engagement. Paper presented at the NAEYC.
- Blum, W., Galbraith, P. L., Henn, H-W. and Niss, M. (Eds) *Applications and Modeling in Mathematics Education*, ICMI Study 14. Amsterdam: Springer.
- Brosterman, N. (1997). *Inventing kindergarten*. New York: Harry N. Abrams.
- Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.
- Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Cobb and Moore, "Mathematics, Statistics, and Teaching," *Amer. Math. Monthly* 104(9), pp. 801-823, 1997.
- Confrey, J., "Tracing the Evolution of Mathematics Content Standards in the United States: Looking Back and Projecting Forward." K12 Mathematics Curriculum Standards conference proceedings, February 5-6, 2007.
- Conley, D.T. *Knowledge and Skills for University Success*, 2008.
- Conley, D.T. *Toward a More Comprehensive Conception of College Readiness*, 2007.
- Cuoco, A., Goldenberg, E. P., and Mark, J., "Habits of Mind: An Organizing Principle for a Mathematics Curriculum," *Journal of Mathematical Behavior*, 15(4), 375-402, 1996.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.
- Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally* (Seventh ed.). Boston: Allyn and Bacon.
- Ginsburg, A., Leinwand, S., and Decker, K., "Informing Grades 1-6 Standards Development: What Can Be Learned from High-Performing Hong Kong, Korea, and Singapore?" American Institutes for Research, 2009.
- Ginsburg et al., "What the United States Can Learn From Singapore's World-Class Mathematics System (and what Singapore can learn from the United States)," American Institutes for Research, 2005.
- Ginsburg et al., "Reassessing U.S. International Mathematics Performance: New Findings from the 2003 TIMMS and PISA," American Institutes for Research, 2005.
- Ginsburg, H. P., Lee, J. S., & Stevenson-Boyd, J. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, 22(1), 1-24.

- Harel, G., "What is Mathematics? A Pedagogical Answer to a Philosophical Question," in R. B. Gold and R. Simons (eds.), *Current Issues in the Philosophy of Mathematics from the Perspective of Mathematicians*. Mathematical Association of America, 2008.
- Henry, V. J., & Brown, R. S. (2008). First-grade basic facts: An investigation into teaching and learning of an accelerated, high-demand memorization standard. *Journal for Research in Mathematics Education*, 39, 153-183.
- Howe, R., "From Arithmetic to Algebra."
- Howe, R., "Starting Off Right in Arithmetic," <http://math.arizona.edu/~ime/2008-09/MIME/BegArith.pdf>.
- Jordan, N. C., Kaplan, D., Ramineni, C., and Locuniak, M. N., "Early math matters: kindergarten number competence and later mathematics outcomes," *Dev. Psychol.* 45, 850-867, 2009.
- Kader, G., "Means and MADS," *Mathematics Teaching in the Middle School*, 4(6), 1999, pp. 398-403.
- Kilpatrick, J., Mesa, V., and Sloane, F., "U.S. Algebra Performance in an International Context," in Loveless (ed.), *Lessons Learned: What International Assessments Tell Us About Math Achievement*. Washington, D.C.: Brookings Institution Press, 2007.
- Leinwand, S., and Ginsburg, A., "Measuring Up: How the Highest Performing State (Massachusetts) Compares to the Highest Performing Country (Hong Kong) in Grade 3 Mathematics," American Institutes for Research, 2009.
- Niss, M., "Quantitative Literacy and Mathematical Competencies," in *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, Madison, B. L., and Steen, L.A. (eds.), National Council on Education and the Disciplines. Proceedings of the National Forum on Quantitative Literacy held at the National Academy of Sciences in Washington, D.C., December 1-2, 2001.
- Pratt, C. (1948). *I learn from children*. New York: Simon and Schuster.
- Reys, B. (ed.), *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?* IAP-Information Age Publishing, 2006.
- Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.
- Schmidt, W., Houang, R., and Cogan, L., "A Coherent Curriculum: The Case of Mathematics," *American Educator*, Summer 2002, p. 4.
- Schmidt, W.H., and Houang, R.T., "Lack of Focus in the Intended Mathematics Curriculum: Symptom or Cause?" in Loveless (ed.), *Lessons Learned: What International Assessments Tell Us About Math Achievement*. Washington, D.C.: Brookings Institution Press, 2007.
- Steen, L.A., "Facing Facts: Achieving Balance in High School Mathematics," *Mathematics Teacher*, Vol. 100. Special Issue.
- Wu, H., "Fractions, decimals, and rational numbers," 2007, <http://math.berkeley.edu/~wu/> (March 19, 2008).
- Wu, H., "Lecture Notes for the 2009 Pre-Algebra Institute," September 15, 2009.
- Wu, H., "Preservice professional development of mathematics teachers," <http://math.berkeley.edu/~wu/pspd2.pdf>.
- Massachusetts Department of Education. Progress Report of the Mathematics Curriculum Framework Revision Panel, Massachusetts Department of Elementary and Secondary Education, 2009.
- [www.doe.mass.edu/boe/docs/0509/item5\\_report.pdf](http://www.doe.mass.edu/boe/docs/0509/item5_report.pdf).
- ACT College Readiness Benchmarks™
- ACT College Readiness Standards™
- ACT National Curriculum Survey™
- Adelman, C., *The Toolbox Revisited: Paths to Degree Completion From High School Through College*, 2006.
- Advanced Placement Calculus, Statistics and Computer Science Course Descriptions. May 2009, May 2010.* College Board, 2008.
- Aligning Postsecondary Expectations and High School Practice: The Gap Defined* (ACT: Policy Implications of the ACT National Curriculum Survey Results 2005-2006).
- Condition of Education, 2004: Indicator 30, Top 30 Postsecondary Courses*, U.S. Department of Education, 2004.
- Condition of Education, 2007: High School Course-Taking*. U.S. Department of Education, 2007.
- Crisis at the Core: Preparing All Students for College and Work*, ACT.
- Achieve, Inc., Florida Postsecondary Survey, 2008.
- Golfin, Peggy, et al. CNA Corporation. *Strengthening Mathematics at the Postsecondary Level: Literature Review and Analysis*, 2005.
- Camara, W.J., Shaw, E., and Patterson, B. (June 13, 2009). *First Year English and Math College Coursework*. College Board: New York, NY (Available from authors).
- CLEP Precalculus Curriculum Survey: Summary of Results. The College Board, 2005.
- College Board Standards for College Success: Mathematics and Statistics. College Board, 2006.
- Miller, G.E., Twing, J., and Meyers, J. "Higher Education Readiness Component (HERC) Correlation Study." Austin, TX: Pearson.
- On Course for Success: A Close Look at Selected High School Courses That Prepare All Students for College and Work*, ACT.
- Out of Many, One: Towards Rigorous Common Core Standards from the Ground Up*. Achieve, 2008.
- Ready for College and Ready for Work: Same or Different?* ACT.
- Rigor at Risk: Reaffirming Quality in the High School Core Curriculum, ACT.
- The Forgotten Middle: Ensuring that All Students Are on Target for College and Career Readiness before High School*, ACT.
- Achieve, Inc., Virginia Postsecondary Survey, 2004.
- ACT Job Skill Comparison Charts.
- Achieve, Mathematics at Work, 2008.
- The American Diploma Project Workplace Study*. National Alliance of Business Study, 2002.
- Carnevale, Anthony and Desrochers, Donna. *Connecting Education Standards and Employment: Course-taking Patterns of Young Workers*, 2002.
- Colorado Business Leaders' Top Skills, 2006.
- Hawai'i Career Ready Study: access to living wage careers from high school*, 2007.
- States' Career Cluster Initiative. *Essential Knowledge and Skill Statements*, 2008.
- ACT WorkKeys Occupational Profiles™.
- Program for International Student Assessment (PISA), 2006.
- Trends in International Mathematics and Science Study (TIMSS), 2007.

International Baccalaureate, Mathematics Standard Level, 2006.

University of Cambridge International Examinations: General Certificate of Secondary Education in Mathematics, 2009.

EdExcel, General Certificate of Secondary Education, Mathematics, 2009.

Blachowicz, Camille, and Fisher, Peter. "Vocabulary Instruction." In *Handbook of Reading Research*, Volume III, edited by Michael Kamil, Peter Mosenthal, P. David Pearson, and Rebecca Barr, pp. 503-523. Mahwah, NJ: Lawrence Erlbaum Associates, 2000.

Gándara, Patricia, and Contreras, Frances. *The Latino Education Crisis: The Consequences of Failed Social Policies*. Cambridge, Ma: Harvard University Press, 2009.

Moschkovich, Judit N. "Supporting the Participation of English Language Learners in Mathematical Discussions." *For the Learning of Mathematics* 19 (March 1999): 11-19.

Moschkovich, J. N. (in press). Language, culture, and equity in secondary mathematics classrooms. To appear in F. Lester & J. Lobato (ed.), *Teaching and Learning Mathematics: Translating Research to the Secondary Classroom*, Reston, VA: NCTM.

Moschkovich, Judit N. "Examining Mathematical Discourse Practices," *For the Learning of Mathematics* 27 (March 2007): 24-30.

Moschkovich, Judit N. "Using Two Languages when Learning Mathematics: How Can Research Help Us Understand Mathematics Learners Who Use Two Languages?" *Research Brief and Clip*, National Council of Teachers of Mathematics, 2009 [http://www.nctm.org/uploadedFiles/Research\\_News\\_and\\_Advocacy/Research/Clips\\_and\\_Briefs/Research\\_brief\\_12\\_Using\\_2.pdf](http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf). (accessed November 25, 2009).

Moschkovich, J.N. (2007) Bilingual Mathematics Learners: How views of language, bilingual learners, and mathematical communication impact instruction. In Nasir, N. and Cobb, P. (eds.), *Diversity, Equity, and Access to Mathematical Ideas*. New York: Teachers College Press, 89-104.

Schlepppegrell, M.J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23:139-159.

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.34 (a). (2004).

Individuals with Disabilities Education Act (IDEA), 34 CFR §300.39 (b)(3). (2004).

Office of Special Education Programs, U.S. Department of Education. "IDEA Regulations: Identification of Students with Specific Learning Disabilities," 2006.

Thompson, S. J., Morse, A.B., Sharpe, M., and Hall, S., "Accommodations Manual: How to Select, Administer and Evaluate Use of Accommodations and Assessment for Students with Disabilities," 2nd Edition. Council of Chief State School Officers, 2005.